Lecture Notes on ΛCDM Cosmology: Is the Big Bang Still Cool?

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1 Introduction

ΛCDM Cosmology is the Big Bang with addition of dark matter and dark energy. The Big Bang has been the standard model of cosmology since the 1964 discovery of the Cosmic Microwave Background (CMB) radiation by Arno Penzias and Robert Wilson. Λ is the term for dark energy in Einstein's field equations. The CDM part of ΛCDM cosmology stands for Cold Dark Matter. Dark energy and dark matter along with ordinary (baryonic) matter provide the mass-energy density to make the spatial geometry of the Universe "flat", or Euclidean, on the largest scales. [1, 2, 3, 4]

Dark matter and dark energy may be the most controversial and interesting features of ΛCDM Cosmology, so we'll look at the dark side first.

2 Dark Matter

Dark matter was first proposed in 1933 by Fritz Zwicky, based on motions of galaxies in the Coma Cluster. Zwicky found the galaxies were moving too fast for the gravitational forces he could account for by observing light from those galaxies. He calculated the amount of dark matter in the cluster should be about 5 times the amount of ordinary matter.

This did not make a big splash at the time. However, in the 1970's, Vera Rubin found the same ratio of dark matter to ordinary matter in her studies of the velocities of stars orbiting a galaxy at various distances from the center. Later work extended the measurements to velocities of hydrogen gas orbiting galaxies at larger distances. However, it remains a mystery as to what the dark matter could be. As Andrew Liddle [1] explained,

> Although the evidence for dark matter is regarded by most as pretty much overwhelming, there is no consensus as to what form it takes.

Many ideas for the composition of dark matter have been proposed. [1] These include fundamental particles that have not yet been observed, and larger objects that are equally unfamiliar. To fit data for the formation of structure in the Universe, the dark matter has to be "cold", that is, non-relativistic. This rules out neutrinos of the types we know about. Also there is a limit to the amount of ordinary (baryonic) matter that could have been present in the early Universe during the time of nucleosynthesis, when primordial helium was formed. This rules out large populations of brown dwarf stars and planets. Other proposals are:

1. Neutrinos, if there is an unknown type of heavy neutrino
2. Supersymmetric particles or Weakly Interacting Massive Particles (WIMPs)
3. Primordial black holes, formed in the very early Universe before nucleosynthesis
4. MAssive Compact Halo Objects (MACHOs) of unknown composition.

So far, nobody has been able to detect any of these.

3 Alternative to Dark Matter: Modified Newtonian Dynamics

In 1983, Mordehai Milgrom proposed an alternate explanation for the velocities of galaxies in the Coma Cluster and stars in galaxies. He suggested that at large distances, the gravitational force falls off more slowly than what we calculate from Newton's law of gravity (Einstein's theory of general relativity gives the same answer in this regime). He called his theory MOdified Newtonian Dynamics, or MOND, and decided the change form Newtonian to MOND gravity happens around a critical value of gravitational acceleration, $a_0 \approx 1.2 \times 10^{-10}\mathrm{m/s}^2$. For Newtonian gravity, the acceleration due to a spherically symmetric source mass is:

$$a = \frac{GM}{r^2} \tag{1}$$

where G = gravitational constant, M = source mass, r = distance from source mass. For MOND, at accelerations much less than a_0, the acceleration due to the same source mass is:

$$a = \frac{a_0 r_M}{r} \tag{2}$$

The factor r_M is supposed to be analogous to the Schwarzschild radius. It is defined by:

$$r_M = \sqrt{\frac{GM}{a_0}} \tag{3}$$

so that the acceleration may also be written as:

$$a = \frac{\sqrt{a_0 GM}}{r} \tag{4}$$

We can calculate r_M for the sun from Equation (3) as:

$$r_M = \sqrt{\frac{6.67 \times 10^{-11}\mathrm{m}^3/(\mathrm{kg\ s})^2 \times 2 \times 10^{30}\mathrm{kg}}{1.2 \times 10^{-10}\mathrm{m/s}^2}} = 1.05 \times 10^{15}\mathrm{m} \tag{5}$$

This is about one tenth of a light year, or 500 times the distance to where Voyager 1 is now. Clearly, this is out of range of our exploration.

A number of researchers have offered modified versions of MOND, but there is still no theoretical justification; it is just curve-fitting. Further, data from other observations of gravitational lensing and anisotropies in the Cosmic Microwave Background favor dark matter. However, research continues on MOND. Two papers from 2023 analyzed orbits of wide binary stars, and came to opposite conclusions about MOND [5, 6].

It is theoretically possible to measure such low gravitational accelerations in the lab, using a Cavendish-type experiment. A group in Vienna reported results using accelerations in the range $1.8 \times 10^{-10}\mathrm{m/s}^2$ to $9.8 \times 10^{-10}\mathrm{m/s}^2$. They discussed possibilities to measure even lower accelerations with the goal of advancing quantum gravity research, and mentioned testing MOND as an extra option.

If MOND is ultimately ruled out, the search for dark matter will take on extra significance.

4 Dark Energy

From 1964 until 1998, it seemed obvious to everyone that gravity would slow down the expansion of the Universe. The big question was, how much would it slow down? Enough to recollapse? That's what the High-Z Supernova Search Team of astronomers and the Supernova Cosmology Project group of physicists decided to find out in the 1990's. Both groups found, to their surprise, the expansion of the Universe appears to be accelerating. Both groups announced results in 1998, and their leaders won the Nobel Prize for Physics in 2011.

The term "High-Z" refers to the redshift, z, which is a function of velocity. Both research teams used Type Ia supernovas as their standard candles to measure distance. Both the expansion rate and the acceleration rate are small, by astronomical standards. When Einstein developed his theory of general relativity in 1915, he assumed the Universe was static, and this was not far from the standard model today.

Einstein's field equations use the energy-momentum tensor as the source of gravitational attraction. This tensor includes energy in the form of mass, plus every other form. He considered including an energy term for empty space, but thought it should be zero. Then he put in a term to make the Universe static, then took it out when he learned about Hubble's observations. Today, astrophysicists assume there is a small energy density of empty space that causes the accelerated expansion of the Universe. This energy of empty space is called "dark energy", in analogy to dark matter. It is characterized mathematically by putting the term Λ back in the general relativity field equations:

$$R^{\mu\nu} = \frac{1}{2} g^{\mu\nu} R + \Lambda g_{\mu\nu} = \left(\frac{8\pi G}{c^4} \right) T^{\mu\nu} \tag{6}$$

Dark energy density is small, but there is a lot of empty space in the Universe. Dark energy contributes about 70% of the total mass-energy of the Universe, dark matter about 25%, and ordinary matter the remaining 5%. This is the right amount to make the spatial geometry of the Universe "flat", or Euclidean, as I will describe in the next section.

5 The Flatness Problem

Euclidean geometry is the ordinary geometry we learn in high school. It works for our three-dimensional space on scales as large as we can reach on Earth, and in calculating the trajectories of spacecraft through the solar system. On the largest scales of the Universe, the spatial geometry could be different; it could be "curved", while Euclidean geometry is "flat". However, it can't be too curved, because flat Euclidean geometry is an excellent approximation at least on small scales.

Einstein's field equations indicate that if the Universe starts with zero curvature, then it will always have zero curvature. However, if it starts with non-zero curvature, then curvature will increase with time. In order for our Universe to be at least approximately flat today, it had to start remarkably close to flat.

Alan Guth's theory of inflation is the accepted explanation for how the Universe got started remarkably close to a perfect flat geometry. Inflation is a big topic, beyond the scope of this paper. It was finished by approximately 10^{-34} seconds after the Big Bang, so it does not affect most of what we are considering here. It does explain how the cosmic microwave background (CMB) radiation looks the same from all directions, even though it came from parts of the Universe that have not been in contact with each other since before the time of inflation.

6 More about Those Red Shifts

Hubble's discovery that galactic red shifts are proportional to distance was the first clear evidence for the Big Bang. The red shifts are interpreted as Doppler shifts due to velocity, and the Hubble law is:

$$v = H_0 d \tag{7}$$

where v = velocity, H_0 = the Hubble parameter at the present time, and d = distance.

The Hubble constant is about 70 km/(s Mpc). Converting to more familiar units,

$$H = \frac{70\text{km}}{\text{s Mpc}} \times \frac{1\text{Mpc}}{3.26 \times 10^6\text{light years}} \times \frac{1\text{light year}}{9.461 \times 10^{15}\text{m}} \times \frac{10^3\text{m}}{1\text{km}} = \frac{2.3\text{m/s}}{10^{18}\text{m}} \tag{8}$$

That is, you have to go a quintillion meters away (621 trillion miles) to get a velocity of 2.3 m/s (5.1 miles/hour).

Astrophysicists tell us the expansion of the Universe affects only the distances between clusters of galaxies; it does not expand or tear apart galaxies or even clusters. Any bound system stays bound: galaxies and clusters are gravitationally bound. Solid objects are bound by chemical forces which are electromagnetic in origin, so a spacecraft in intergalactic space would not expand either. If space expanded uniformly, we would never observe it because our bodies and measuring instruments would expand along with the rest of the Universe. Also, we don't think the galaxies have been pushed away from each other by some kind of explosion, because the expansion of the Universe began long before galaxies formed. We will look at evidence for this in later sections.

Here's what Misner, Thorne, and Wheeler had to say about the expanding Universe in their classic book "Gravitation" [8], p. 719:

> All distances between spatial grid points (fluid world lines) expand by the same factor $a(t)$, leaving the hypersurface unchanged. This is a consequence of homogeneity and isotropy; and it is precisely true only if the model universe is precisely homogeneous and isotropic.
>
> Of all the disturbing implications of "the expansion of the universe," none is more upsetting to many a student on first encounter than the nonsense of this idea. The universe expands, the distance between one cluster of galaxies and another cluster expands, the distance between sun and earth expands, the length of a meter stick expands, the atom expands? Then how can it make any sense to speak of any expansion at all? Expansion relative to what? Expansion relative to nonsense! Only later does he realize that the atom does not expand, the meter stick does not expand, the distance between sun and earth does not expand. Only distances between clusters of galaxies and greater distances are subject to the expansion. Only at this gigantic scale of averaging does the notion of homogeneity make sense. Not so at smaller distances. No model more quickly illustrates the actual situation than a rubber balloon with pennies affixed to it, each by a drop of glue. As the balloon is inflated the pennies increase their separation one from another but not a single one of them expands!

The Andromeda galaxy is actually moving toward us at 300 km/s, since it is gravitationally bound in our Local Group of galaxies. It is also 2.5 million light years away, which is less than 1 Mpc, so the Hubble expansion would send it away at less than 70 km/s if it were not bound.

Einstein did not know about other galaxies in 1915, because they showed up only as fuzzy patches of light in the telescopes of that time. Five years later, the astronomical "Great Debate" between Harlow Shapley and Heber Curtis established the idea that these fuzzy patches, or nebula, could be other galaxies. Throughout the 1920's, astronomers interpreted more and more nebula as galaxies, and noted their spectra were usually red-shifted. In 1929, Hubble published his first paper indicating the velocity of recession is proportional to distance. This paper sparked much more work on the Big Bang.

7 Alternate Interpretation of Red Shifts: Segal's Chronometric Cosmology

We do not have direct evidence that galaxies are moving away from us; we only have the red shifts of their spectra. Irving Segal interpreted these red shifts to be a result of curvature of the Universe, which he considered to be a 3-sphere. He developed a theory called chronometric cosmology based on group theory math from quantum mechanics.[9] In this cosmology, the Universe is infinitely old. His equation for red shift as a function of distance is:

$$z = \tan^2(r/2R) \tag{9}$$

where z = red shift, r = distance, and R = radius of the Universe. R is the only free parameter in the theory, and is approximately 500 million light years [10]. Segal's work has not been widely accepted, but is published in reputable journals. I am curious whether something similar could be an alternative to dark energy. Or maybe future work will add a geometrical correction to ΛCDM cosmology.

8 Thermodynamics

In the 19th Century, engineers studying efficiency of heat engines discovered key concepts of how the Universe works. Thermodynamics is used in many fields, from chemical engineering to astrophysics. The first two laws of thermodynamics, as stated by Rudolph Clausius, are:

1. "Die Energie der Welt ist constant.

2. Die Entropie der Welt strebt einem Maximum zu."

Here's a standard translation, with the 3rd law added:

The Energy of the Universe is constant.
The Entropy of the Universe tends toward a maximum.
It is impossible to achieve the minimum entropy at absolute zero.

And here's a more popular translation:

You can't win.
You can't break even.
You can't get out of the game.

Entropy is a measure of how much energy is not available to do work. It can be calculated from classical thermodynamics as:

$$S = \oint \frac{Q}{T} \tag{10}$$

where S = entropy, Q = heat transferred, T = temperature. Entropy can also be defined from statistical mechanics as:

$$S = k \ln W \tag{11}$$

where k is Boltzmann's constant and W is the number of configurations (microstates equivalent to a macrostate). For cosmology, the primary significance of thermodynamics is the universal increase of entropy, until equilibrium is reached. We assume the Universe is an isolated system, since there is nothing outside for it to interact with. Any isolated system will reach equilibrium in finite time. Since our Universe is clearly not at equilibrium, it must have started in a low-entropy state at a finite time in the past. The Big Bang is an excellent explanation of where the initial low entropy came from; the gravitational potential energy has minimum entropy for a system with a uniform density of mass-energy. Such a system will spontaneously clump into structures such as galaxies and stars.

The physics literature has a well-known quote from Arthur Eddington [11]:

> "The law that entropy always increases holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations - then so much the worse for Maxwell's equations. If it is found to be contradicted by observation - well, these experimentalists do bungle things sometimes. But if your theory is found to be against the Second Law of Thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation."

Of course, Eddington could have been wrong. It may be that we need to update our theory of thermodynamics, but the current theory has been so useful for so many applications that it is probably the last thing we should consider changing.

9 Helium Abundance

Helium was discovered in the sun, from its spectral lines, before it was found on Earth. Spectroscopic observations of other stars and of interstellar gas indicate that helium makes up about 25% of their mass, the rest being mostly hydrogen. Helium is produced by nuclear

Table 1: Abundance of Isotopes in the Universe from Big Bang Nucleosynthesis

Isotope	Observed	Predicted
Helium-4 mass fraction	0.253	0.246
Deuterium per 10^5 protons	2.57	2.67
Helium-3 per 10^5 protons	1.1	1.05
Lithium-7 per 10^7 protons	1.58	4.89

fusion in the cores of stars, but this amount is not usually dispersed into interstellar space; it ends up in burnt-out stars.

The Big Bang accounts for the abundance of helium as follows: as we look back in time, we see the temperature of the Universe increase. Sufficiently far back, the Universe was too hot for protons and neutrons to exist; everything was a soup of quarks, gluons, leptons, and photons. As the Universe cooled, the quarks and gluons formed protons and neutrons. There was period of a few minutes when the temperature was in the range for protons and neutrons to react with each other to form nuclei heavier than hydrogen. Neutrons are unstable; they decay into protons plus electrons and neutrinos. Also tritium is unstable and decays into helium-3. Detailed calculations of nuclear fusion reactions indicate the Universe formed approximately the observed amounts of these stable nuclei. After this time the Universe was too cool for nuclear reactions, but still too hot for atoms to form until about 380,000 years later.

Table 1 shows the relative abundance of isotopes produced from Big Bang nucleosynthesis, observed and calculated [4]. Agreement is excellent for all but Lithium-7, which is the rarest. This calculation depends on the baryon-to-photon ratio, and is an important reason why dark matter can not consist primarily of ordinary baryonic matter, unless it is in the form of primordial black holes that formed before the era of nucleosynthesis.

10 Cosmic Microwave Background

The cosmic microwave background (CMB) radiation is our richest source of data on the Big Bang. We can't see it with our eyes, but in terms of energy density, it is the brighter than all the starlight in the Universe. It gives us a snapshot of the Universe when it was approximately 380,000 years old. For any direction we look, the CMB has the spectrum of a blackbody with temperature about 3 K. The exact temperature has small variations, or anisotropies, in different directions. These are plotted in the familiar image of Figure 1. The data come from three satellites: COBE, WMAP, and Planck. The different colors represent different temperatures, and these differences in temperature were caused by different densities in the early Universe, as matter began to form structure under the influence of its own gravity.

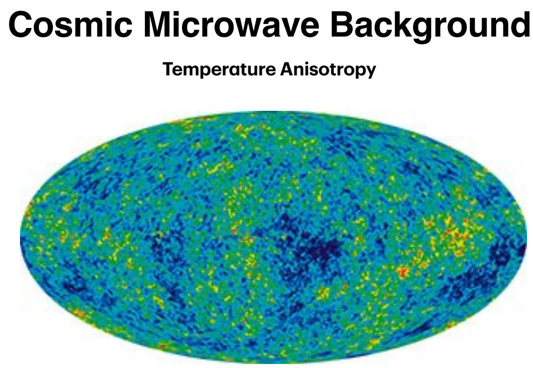

Figure 1: CMB Radiation

The temperature variations look fairly random, but they are not. An expansion in multipole moments, equivalent to a spatial Fourier transform, shows distinct peaks, as in Figure 2.

The peaks can be calculated from the ΛCDM Cosmology model, with input data including the Hubble parameter and the mass density of the Universe. These calculations are complicated, and were started back in the 1970's. In practice, researchers run simulations of the early Universe with a range of input parameters, and see what comes out in terms of the CMB power spectrum and the abundances of isotopes that we looked at in the previous section. The parameter values that give the best fits to all the data are considered correct.

This brings us to the "Hubble tension" [12]. The Hubble constant has been measured by observing the distance and red shifts of galaxies, with a value of 74 in units of km/ (s Mpc). Calculations from the CMB power spectrum indicate the best value should be 67. While these values are close, the difference indicates we still have more to learn about the origins of our Universe.

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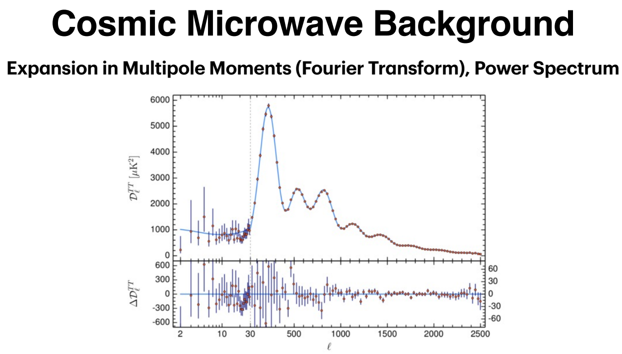

Figure 2: CMB Power Spectrum

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