An Easier Way to Understand Bell's Theorem for Entangled Photons

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Abstract

Bell's theorem demonstrates the properties of quantum entanglement, which are important for both technological and philosophical reasons. This paper presents a simplified way for students and popular physics fans to understand entanglement conceptually.

1 Introduction

Entanglement is a surprising property of quantum systems, such as photons, that can be put to use in quantum computing, telecommunications, and cryptography [Nielsen and Chuang, 2011]. John Bell offered an amusing way to understand this phenomenon in his paper "Bertlmann's Socks and the Nature of Reality", available online [Bell, 1980]. The mathematical description of entanglement is not advanced, but the concepts are so different from ordinary experience that many people, especially popular science fans, remain confused after reading multiple explanations. The time machine story in this paper is a new approach that has been well received in public seminars presented by the author.

The key experiments were done by Alain Aspect, John Clauser, and Anton Zeilinger, who won the 2022 Nobel Prize in Physics for their work on entanglement. The production and measurement of entangled photons are described in [Aspect et al., 1981] [Aspect et al., 1982].

2 Experimental Set-up

Here's an idealized version of the experiment. Put a source of entangled photons between Detector A and Detector B, which can measure the polarization of each photon. The details of how to do this are described in [Aspect et al., 1981]. Each detector has an axis which we designate as "vertical", and a perpendicular axis which we designate as "horizontal". The vertical axis of the detector does not have to be aligned with the vertical direction of the lab; we will take it to be the axis closest to the lab vertical for convenience. Each time a detector measures a photon, it reports the photon is vertical or horizontal in terms of the detector axes.

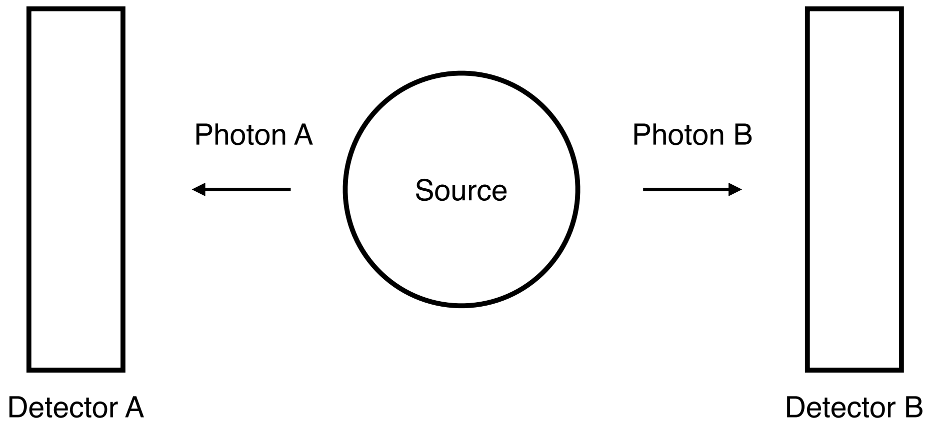

Figure 1: Entangled Photon Experiment

The entangled photon source produces a series of photon pairs. As long as the polarization detector axes on both sides of the source are set at the same angle, both detectors produce the same results. The interesting results come when the detector axes are set at different angles.

The polarization of each pair measured at any angle (same angle for both twins) is found to be a random sequence of vertical and horizontal, with overall half vertical and half horizontal. Quantum theory predicts this, but we can just take it as an experimental fact for the purpose of understanding entanglement.

It looks like the two photons have the same polarization when they leave the source. Bell's Theorem shows us that can't be true. The usual interpretation of Quantum Mechanics tells us the properties of any quantum system, such as the polarization of a photon, are not real until they are measured. The Aspect experiments show us what that means.

I made a random series of 50 Hs and Vs by looking at the digits of pi. Odd digits get V, even digits get H. Suppose we do the entangled photon experiment with both polarization detectors set at 0° to the lab vertical. Then both detectors will measure the same sequence of H's and V's. Let's say "V" represents a vertical photon, and "H" represents a horizontal photon. The sequence of photons detected could look like this:

Run #1: Both detectors at 0°

A: `VHVVVHHVVVHVVVVHVHHHHHHVVHVHVVVHHHHHVVVVHVVVVVVVVH`

B: `VHVVVHHVVVHVVVVHVHHHHHHVVHVHVVVHHHHHVVVVHVVVVVVVVH`

3 Time Machine Story

Now for the idealized experiment, let's suppose we have a time machine. We go back in time and repeat the experiment, with one change. Rotate Detector B to $+22.5°$ from the lab vertical. So the difference in angles is $\Delta\theta = 22.5°$.

Classical electromagnetism tells us that if we send a beam of unpolarized light through two polarizers, then 50% of the beam will go through the first one, and then 85% of the light which goes through the first polarizer will also go through the second one. We calculate this from Amplitude $= \cos 22.5° = 0.924$; Intensity = Amplitude squared $= 0.854$.

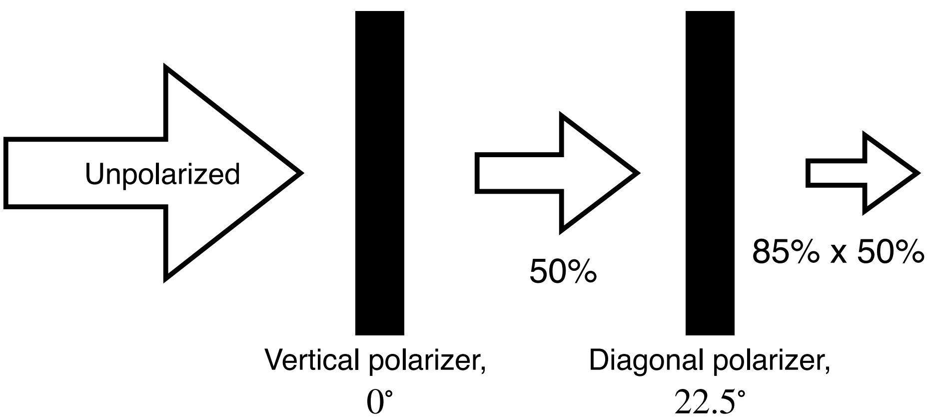

Figure 2: Classical Light Beam and Polarizers

The entangled photon experiment of Figure 1 is a little different, because the source is located between the two photons. Classical electromagnetism tells us only that 50% of the photons will be detected by each detector.

Quantum mechanics tells us more about the entangled photon experiment. It says the results of the two polarization detectors will agree 85% of the time, and disagree 15% of the time. Which pairs will disagree? We don't know; all we can calculate is the statistical result, as in playing dice. We assume Detector A gives the same results as it did the first time, because what we do to Detector B should not affect what happens on the other side.

Let's say the agreements are "hits" and the disagreements are "misses", or mismatches. Here's a possible sequence for our second run, with the misses printed in red:

Run #2: Detector A at 0°, Detector B at +22.5°:

A: VHVVVHHVVVHVVVVHVHHHHHHVVHVHVVVHHHHHVVVVHVVVVVVVVH

B: VHVHVHHVHVHVVVVHVHHHVHHVVVVHVHVHHHHVVVVVHVVVHVVVVH

I have room across the page for 50 letters, and 15% of 50 is 7.5. So I changed seven of the letters in the Detector B line to the opposite polarization: four V's are changed to H, and three H's are changed to V.

Now let's go back in time and reset the polarizers again. Put Detector B back to 0°, and move Detector A to −22.5°. The difference in angles is again $\Delta\theta = 22.5°$. Now we expect Detector B to give us the same results as it did in Run #1, since it is in the same position and we are measuring the same series of photons, thanks to our time machine. This time we expect Detector A to give us approximately 15% misses; that is; 15% of the photons will be measured with polarization different from how they were measured in Run #1. Again, these misses happen at random. Here's a possible sequence for our third run, with eight misses printed in red:

Run #3: Detector A at −22.5°, Detector B at 0°:

A: VHVHVVHVVVHVHVVHVHVHHHVVVHVHVVVHHVHHVVVVHHVVVVHVVH

B: VHVVVHHVVVHVVVVHVHHHHHHVVHVHVVVHHHHHVVVVHVVVVVVVVH

And now for the interesting part. Let's go back in time for one final run, and set Detector A at −22.5° and Detector B at +22.5°. Now we have $\Delta\theta = \Delta\theta_1 + \Delta\theta_2 = 45°$ degrees. Again, we expect the results at each detector to be the same as they were when the detector was set to the same angle previously, since the time machine allows us to measure the same series of photons.

Run #4: Detector A at −22.5°, Detector B at +22.5° in the time travel story:

A: VHVHVVHVVVHVHVVHVHVHHHVVVHVHVVVHHVHHVVVVHHVVVVHVVH

B: VHVHVHHVHVHVVVVHVHHHVHHVVVVHVHVHHHHVVVVVHVVVHVVVVH

Now let's count the number of letters printed in red. It is 8 + 7 = 15, or approximately, 15% + 15% = 30% (since we have only 50 letters for each detector). Each red letter represents a "miss" from either Run #2 or Run #3; that is, the red digits are different from the digit in the same position in Run #1. Sometimes the misses from Run #2 (red) and the misses from Run #3 (red) can occur for the same photon pair, so we have two reds together, and they match. I deliberately made the first red letter in each run match; it is a red H in the fourth position.

So the number of misses in Run #4 is less than or equal to 30%. It can't be more. At least 70% of the photon detections should match when the polarizers are at an angle of 45 degrees to each other.

That's the end of the time travel story.

4 Experimental Results

Now let's return to reality, where we can't do time travel. Also we don't have perfectly efficient detectors. We can still take a lot of data and do a careful mathematical analysis. The statistical results are the same as the results in our time travel idealized experiment for the first three runs (but not Run #4): we get complete agreement when the detectors are at the same angle, and 15% mismatches when the polarizers are 22.5° apart.

Bell's Inequality, in this experiment, is:

$$\text{Mismatches}(\Delta\theta_1) + \text{Mismatches}(\Delta\theta_2) \leq \text{Mismatches}(\Delta\theta_1 + \Delta\theta_2). \tag{1}$$

For our case,

$$\Delta\theta_1 = \Delta\theta_2 = 22.5^\circ \tag{2}$$

and

$$\Delta\theta_1 + \Delta\theta_2 = 45^\circ \tag{3}$$

Bell's Theorem tells us the inequality will be true if the polarization of the photons is objectively and locally real, which is what we expect from common sense. If the polarization of each photon is real when it leaves the source, and the two polarizations are the same, then Bell's inequality must be true.

And now for the surprise. What actually happens when the polarizers are set at an angle of 45° to each other? They agree only half the time. Bell's Inequality is violated experimentally. Here's a possible series for a real life experiment, with more pairs that do not match:

Run #4: Detector A at , Detector B at IN REAL LIFE:

A: HHVHVVHHVVVVHVVHVHVHHHVVVHVHVVHHHVHHVHVVHHVVVVHVVV

B: VVVHVHVVHVHVVHVHVVHHVHHVVVVHVHVHVHHVVVHVHVVVHVVVHH

For classical electromagnetism, consider Figure 2 but with the second polarizer at 45° to the first one. Then we calculate, for light that goes through the first polarizer, only 50% goes through the second one: Amplitude = $\cos 45^\circ = 0.707$; Intensity = Amplitude squared = 0.50.

And from quantum mechanics, we calculate the polarizations in the entanglement experiment should agree 50% of the time. Quantum mechanics clearly gives us the right results, but it does not seem possible. Each photon appears to "know" how both polarizers are set, and what happens to its twin on the other side of the experiment. Einstein called this sort of phenomenon "spooky action at a distance".

To make a stronger case, Aspect et al. did the experiment with very fast electronics that sent each photon to one of two different polarization detectors [Aspect et al., 1982]. That way, the photon could not know which polarization angle would be measured until after it was in flight. Then there was not enough time for either photon to communicate with its twin before it was detected, unless it could send a signal faster than light.

5 Conclusions

What happened here? Many books have been written about the interpretation of Quantum Mechanical calculations [Maudlin, 2019] [Baggott, 1992] [Albert, 1992] [Wheeler and Zurek, 1983]. Our common sense is based on Classical Mechanics, and Quantum Mechanics is different. Albert Einstein wrote a famous 1935 paper, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" [Einstein et al., 1935]. Here he suggested there must be more to quantum theory yet to be discovered, and that extra part will clear up our difficulty in understanding it. He proposed an experiment using entangled particles such as electrons, with measurements of position and momentum. John Bell modified this thought experiment to use electron spin measurements. Alain Aspect modified it further to use photon polarization. Each electron or photon is considered a "system" with properties to be measured.

The logic is the same for each case. To quote from Einstein, "This makes the reality of [properties measured for the second system] depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this." Bell's Theorem tells us that regardless of whether it's reasonable, that's the way reality is.

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